Extension of the Numerical Schemes in an Overset Cartesian Grids Framework

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10th Overset Grid Symposium

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Outline

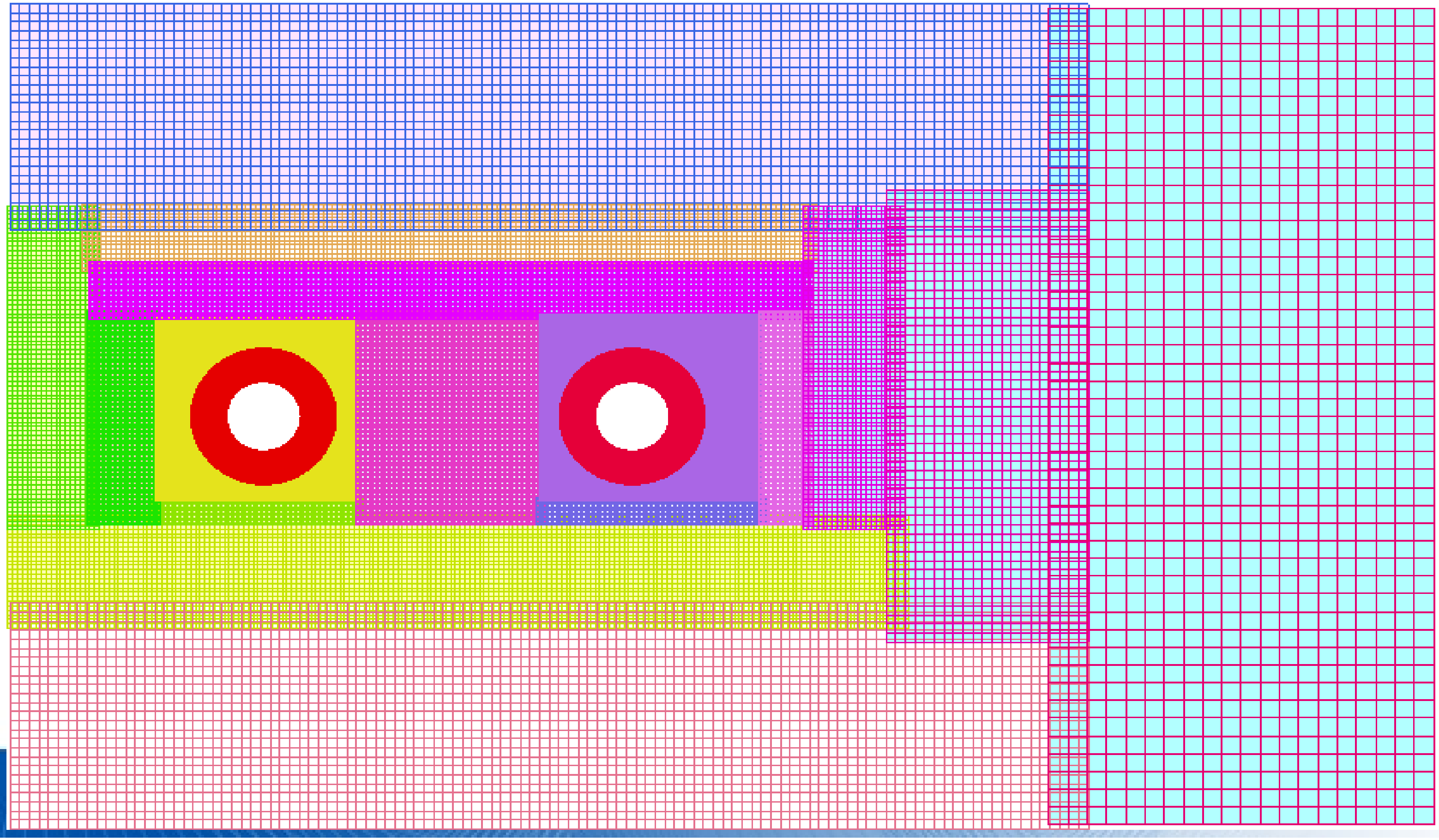
- Overset Cartesian Mesh method
- Adaptation of centered schemes
- Adaptation of upwind schemes
- Conclusion

Overset Cartesian Grids method

- Introduced by R. Meakin (2000)
- Input: body grids
  - Accurate description of geometry
  - Short extension

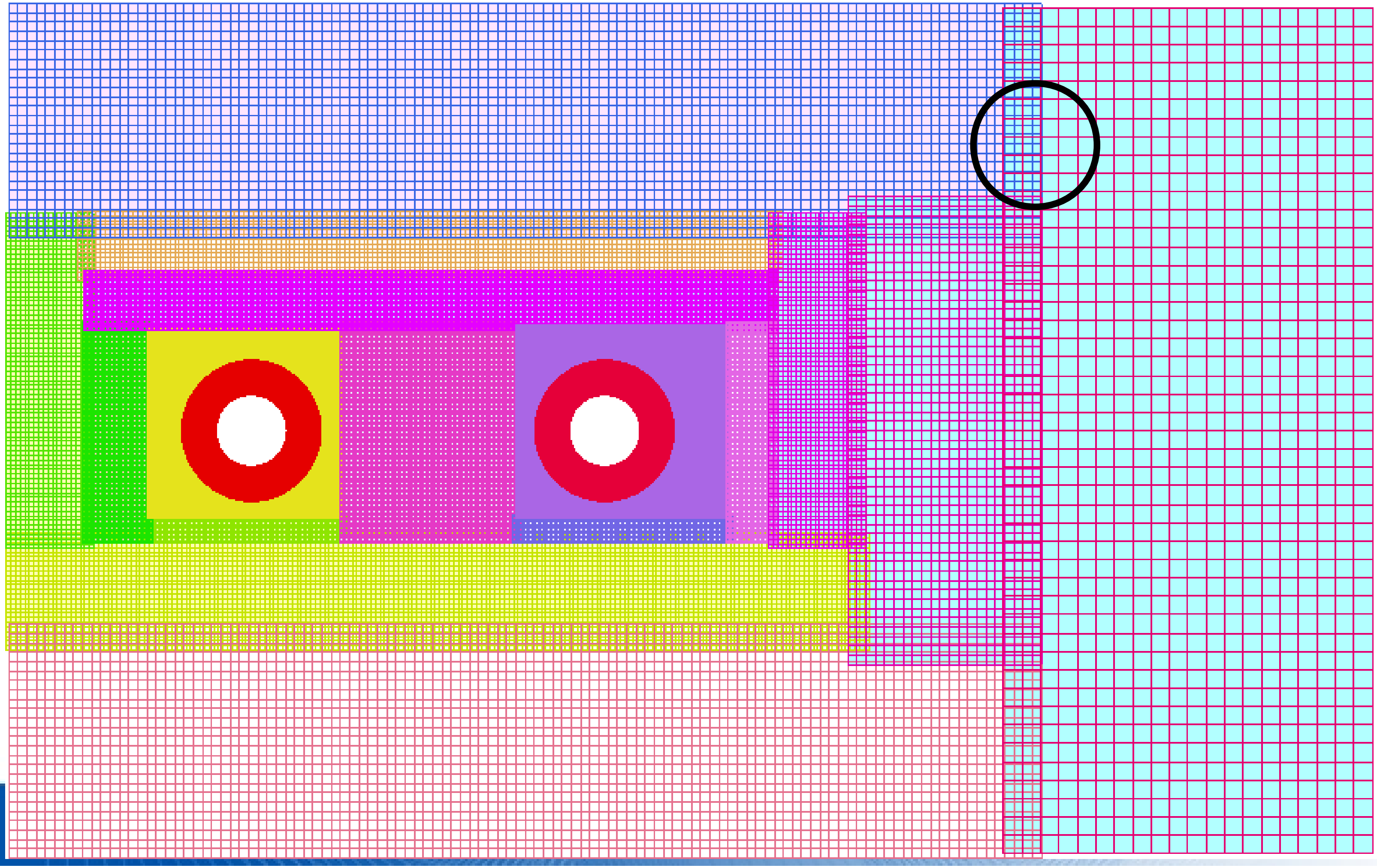
Overset Cartesian Grids Method

- Initial Cartesian mesh generation
  - Set of regular Cartesian grids
  - Finer step near bodies
  - Control of the minimum number of points per grid



Overset Cartesian Grids Method

- Initial Cartesian mesh generation
  - Cartesian Grids overlap each other with minimum overlap

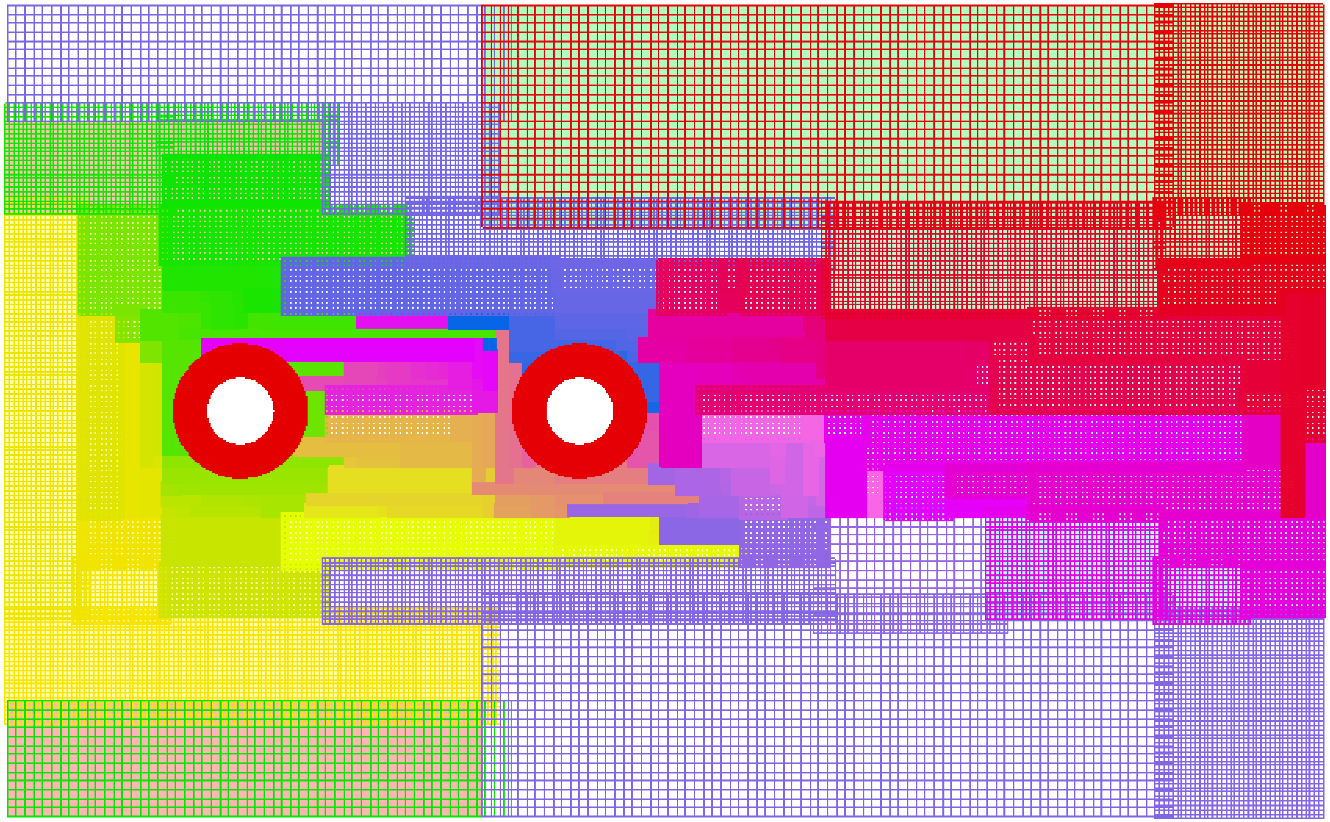


Overset Cartesian Grids Method

- Overlap regions
  - Transfer by interpolation of conservative field
  - Minimum overlap between grids ensured
  - Mesh size coherent between curvilinear body grids and Cartesian grids in overlap regions
  - Matching boundary conditions between Cartesian grids of finest level

Cartesian Overset Grids Method

- Cartesian mesh adaptation
  - Cartesian grids can be regenerated following a criteria evaluated on the previous mesh



Centered schemes

- Starting point:
  - Jameson scheme (order 2)
  - Finite volume formulation on curvilinear structured grids
  - Artificial dissipation

Centered schemes

- Adaptation to regular Cartesian grids:
  - Face normal is constant
  - Cell volume is constant

  → No metric storage
  - The flu $\int_{\partial\Omega} F(w).n \, d\Gamma$ can be simplified
  - Implicit matrix is sparser
  - Interpolation cell search is simple and fast

  → Gain of 40% in CPU and memory

Order 3 centered scheme

- Extension to third order on curvilinear structured grids:

  Introduced by Rezgui, Cinella, Lerat (2001)

  - Third order estimation of flux integral
  - Third order estimation of volume integral
  - Correction of centered approximation of interface fluxes

  → Order 3 on deformed meshes

  → But: requires new local metrics (distance from cell-center to interface center, …)

  → Expensive in memory and CPU time

Order 3 centered scheme

- Extension to third order on curvilinear structured grids:
  - Hypothesis of regular meshes with small rotation between cells
    - Simplification in integral formulae
    - Simplification in Taylor expansions

    → Order 3 on regular meshes only
  - Flux is approximated at interface center by:

$$F_{i+\frac{1}{2},j,k} = [(I - \frac{1}{8}\delta_1^2 + \frac{1}{24}\delta_2^2 + \frac{1}{24}\delta_3^2)\mu_1\phi]_{i+\frac{1}{2},j,k}$$

  - With:

$$(\mu f)_{i+\frac{1}{2}} = \frac{1}{2}(f_i + f_{i+1}), \quad (\delta f)_{i+\frac{1}{2}} = f_{i+1} - f_i$$

$\Phi = F(w).n$

Order 3 centered scheme

- Extension to third order on Cartesian grids:
  - Finite volume formulation reduce to finite difference formulation!
  - Complete scheme reads:

$$\left[w_t + \frac{\delta_1}{\delta x}(I - \frac{1}{6}\delta_1^2)\mu_1 f + \frac{\delta_2}{\delta y}(I - \frac{1}{6}\delta_2^2)\mu_2 g + \frac{\delta_3}{\delta z}(I - \frac{1}{6}\delta_3^2)\mu_3 h\right]_{i,j,k} = 0$$

→ Easy to implement

→ Only 5% CPU overhead than Jameson scheme

Order 3 centered scheme

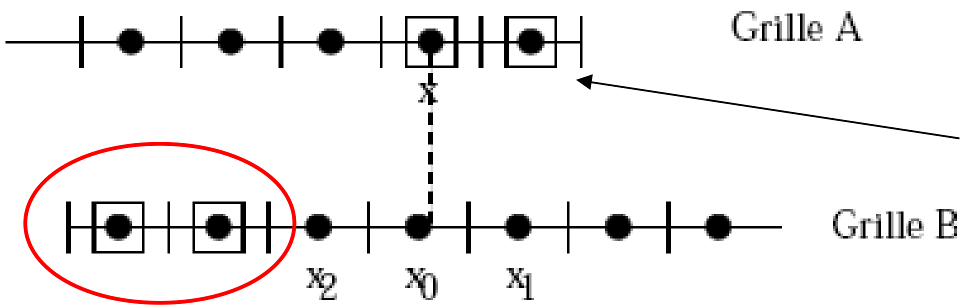
- Numerical dissipation:
    - Same as Jameson scheme:

$$F_{j+\frac{1}{2},k} = (F - D)_{j+\frac{1}{2},k}$$

$$(D)_{j+\frac{1}{2},k} = \rho(\bar{A}_{j+\frac{1}{2},k})(\epsilon_2 \delta_1 w - \epsilon_4 \delta_1^3 w)_{j+\frac{1}{2},k}$$

$O(h^3)$ $\qquad$ $O(h^3)$

Chimera interpolation



- Interpolation through Lagrangian polynomials of degree 2

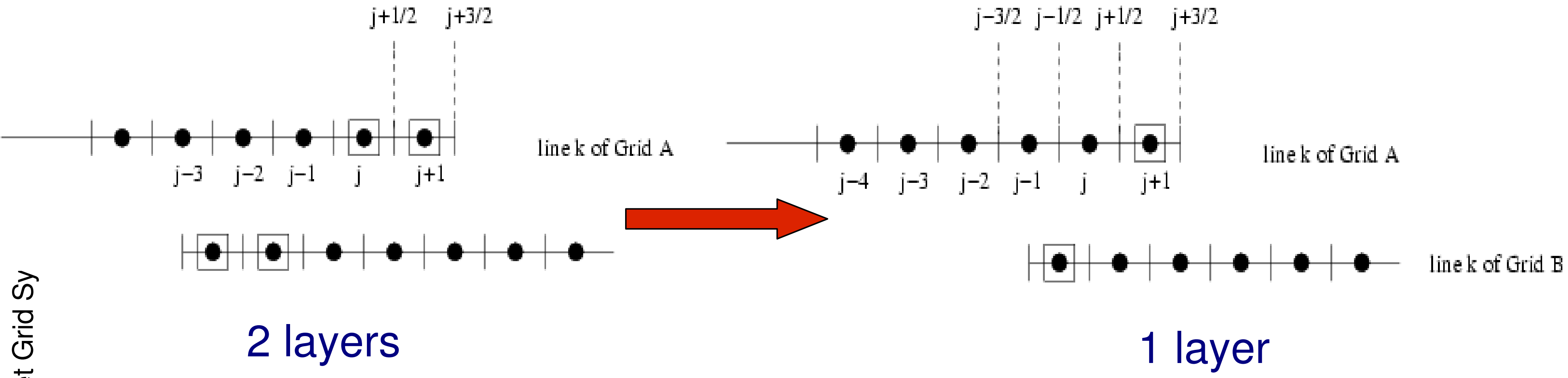
$$P_n(x) = \sum_{i=0}^{n} l_i(x) w(x_i)$$

$$l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}, \qquad 0 \leq i \leq n$$

With n=2

One layer of interpolated cells

- To minimize the overlap constraint:
  - Enable only one layer of interpolation cells
  - Upwind formulae near interpolation boundaries



Order 5 centered scheme

- Only for Cartesian grids
- Finite difference centered scheme
- Artificial dissipation of higher order
- Only 10% CPU overhead than Jameson

$$(w_t)_j + \frac{\delta}{\delta x}\left[\left(I - \frac{1}{6}\delta^2 + \frac{1}{30}\delta^4\right)\mu f\right]_j = \frac{\delta}{\delta x}\left(\frac{1}{60}|A_R|\delta^5 w\right)_j$$

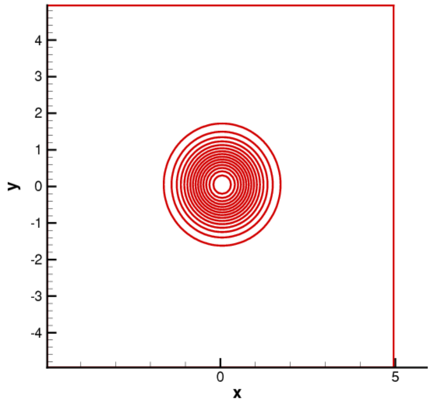
Convective flux

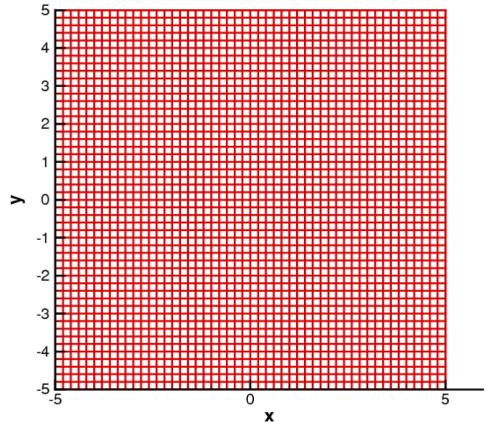
Dissipation

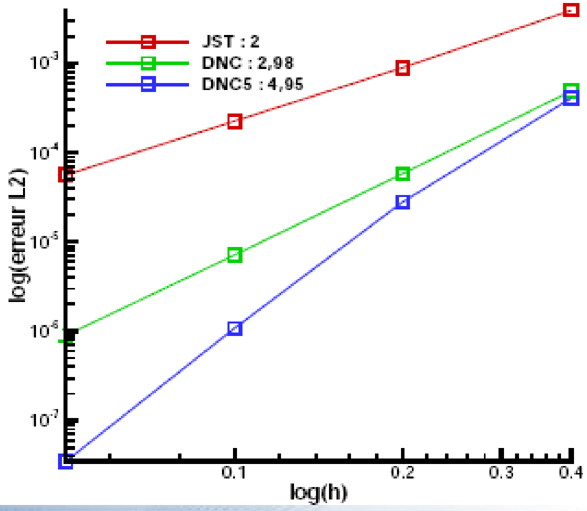
Validation: vortex advection

- Test case: Yee (2000)
- Inviscid flow, M=0.4225
- Regular Cartesian mesh

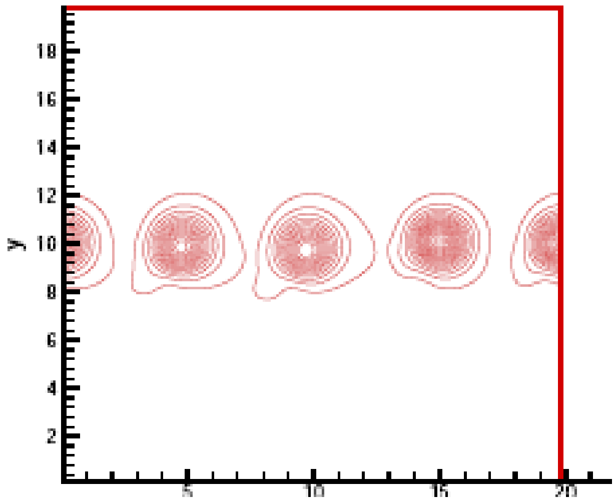
→ Accuracy order is achieved



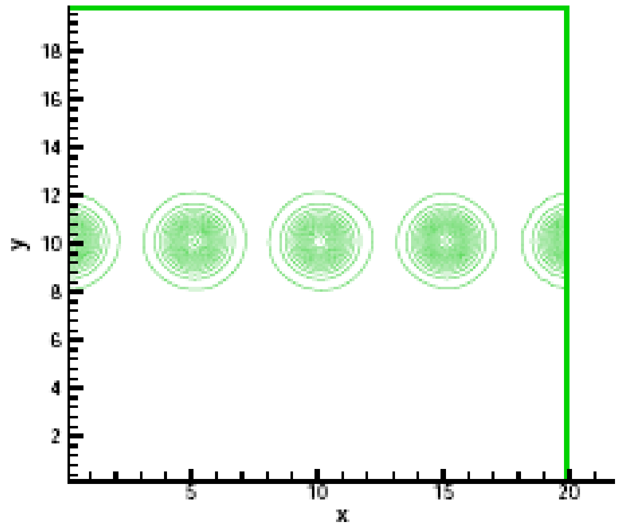




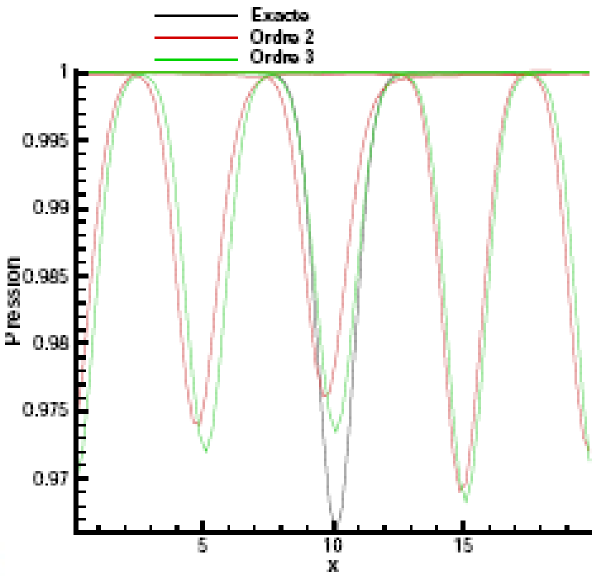
Validation: vortex advection



Order 2

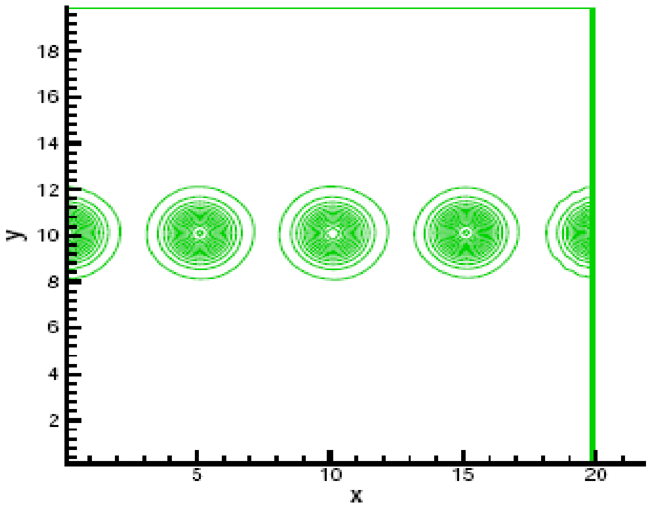


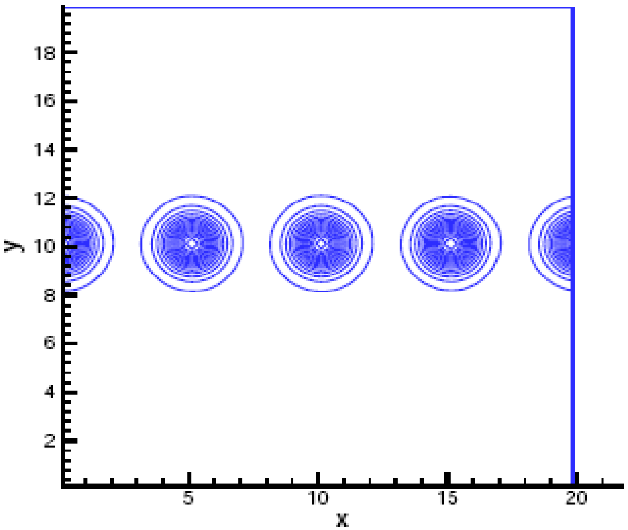
Order 3

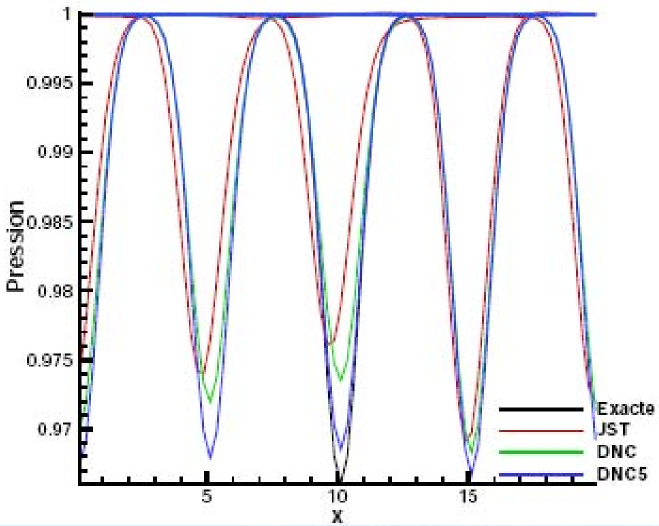


Pressure loss at vortex center after on revolution:
Order 2: 30%
Order 3: 22%

Validation: vortex advection



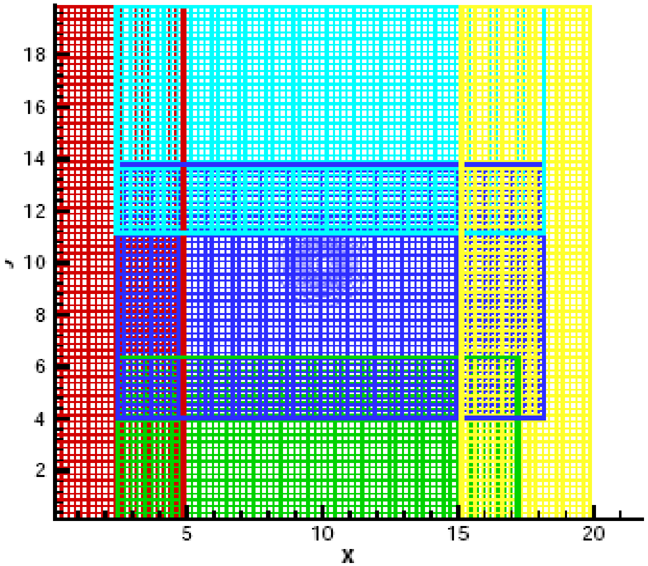


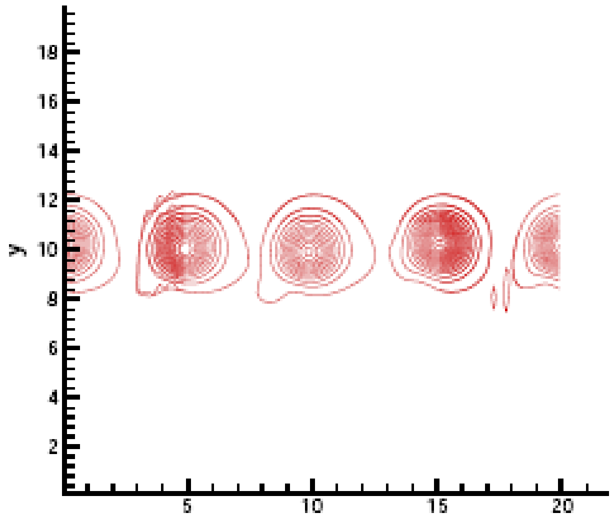


Pressure loss at vortex center after on revolution:
Order 2: 30%
Order 3: 22%
Order 5: 7%

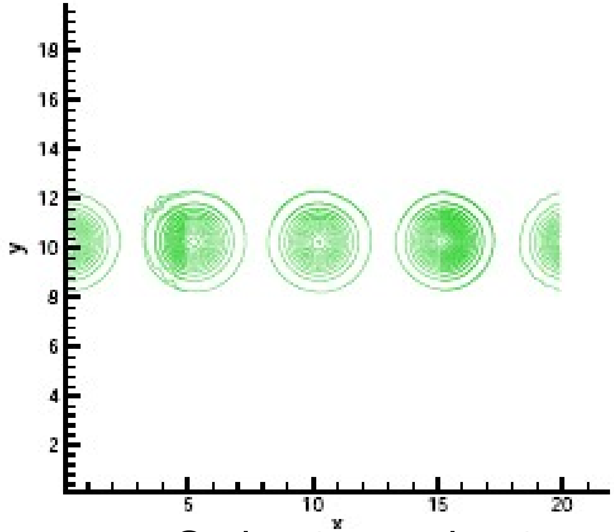
→ Order 5 is necessary for an accurate advection

Validation: vortex advection

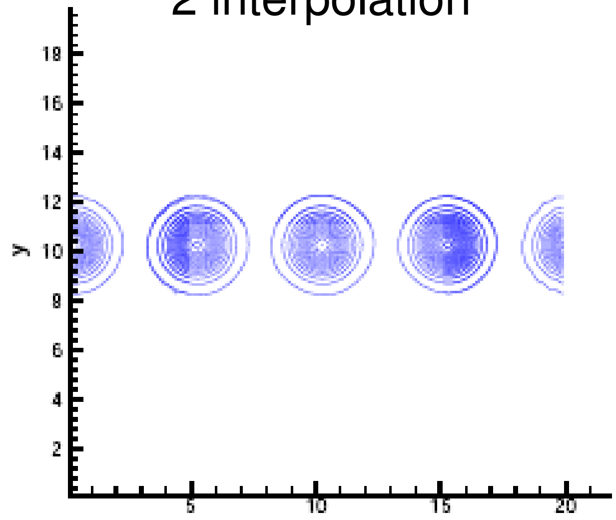




Jameson + order 2 interpolation



Order 3 + order 2 interpolation

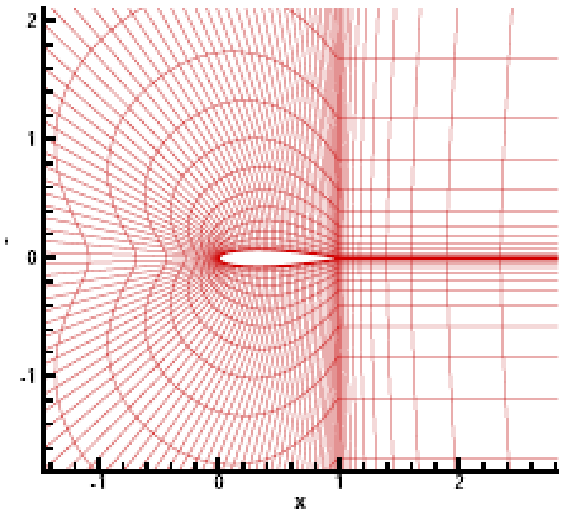


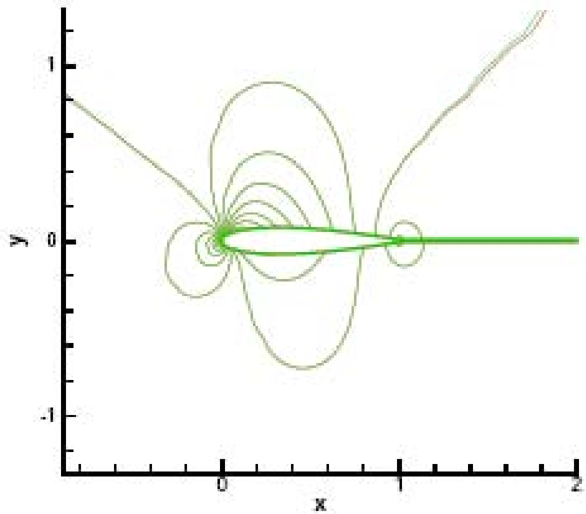
Order 3 + order 3 interpolation

→ Interpolation order must be compatible with scheme order

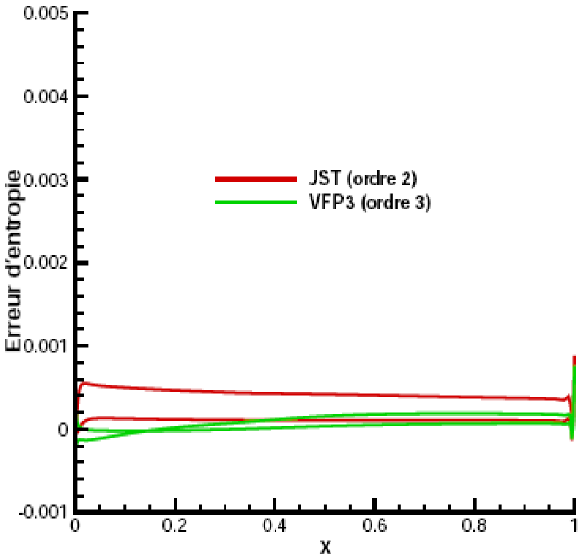
Validation: order of accuracy

- Subsonic flow around a NACA profile, M=0.63, 2 degrees of incidence
- Monobloc curvilinear grid 273x45





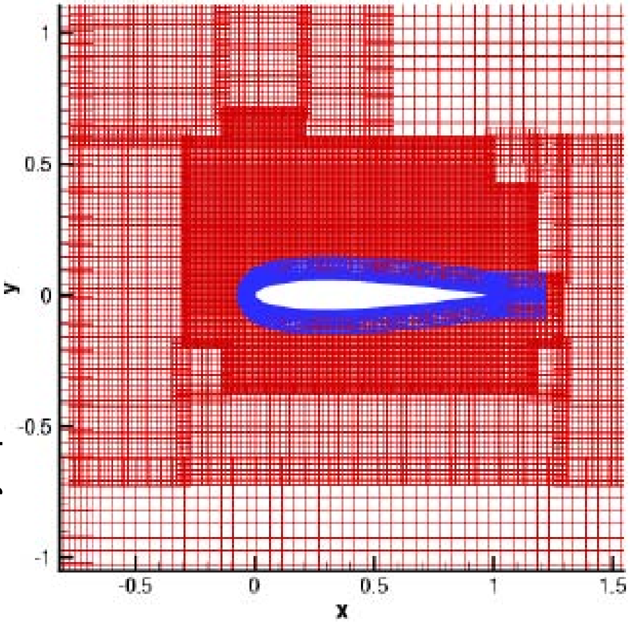
Iso-density lines



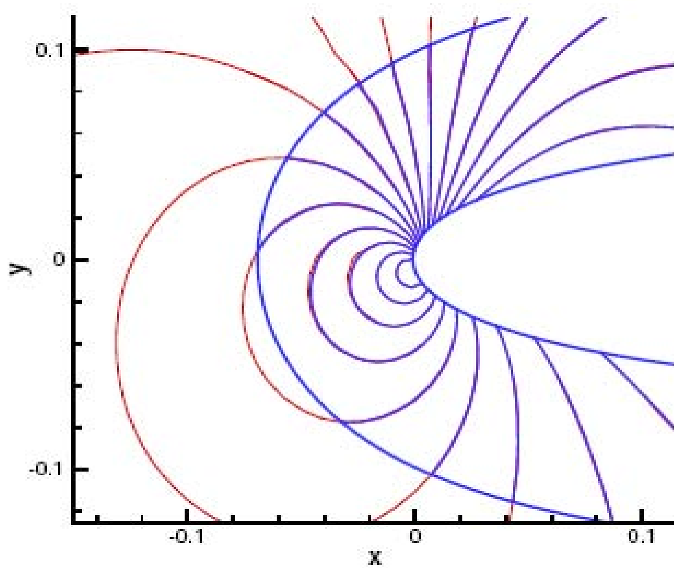
Wall entropy

Validation: order of accuracy

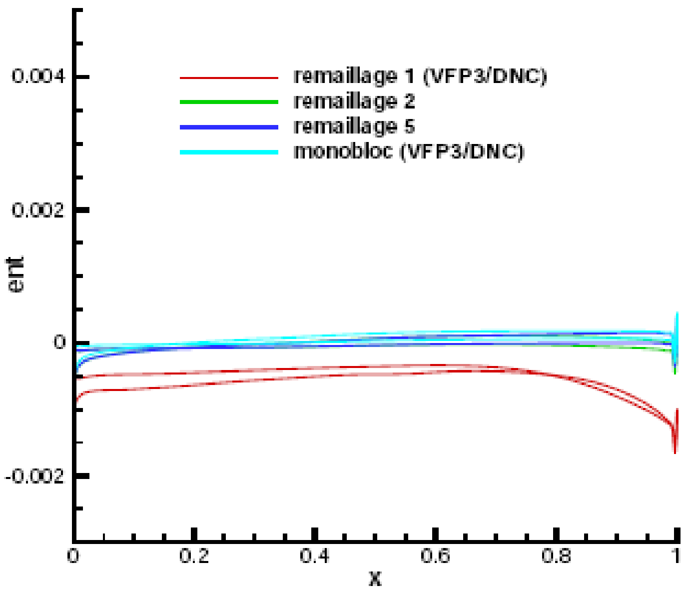
- Generation + 5 mesh adaptations
- Order 3 + order 3 interpolations



Adapted mesh



Iso-density lines

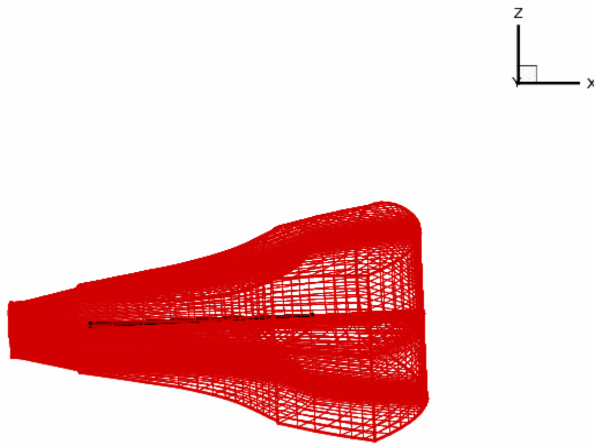


Wall entropy

→ Order 3 is achieved

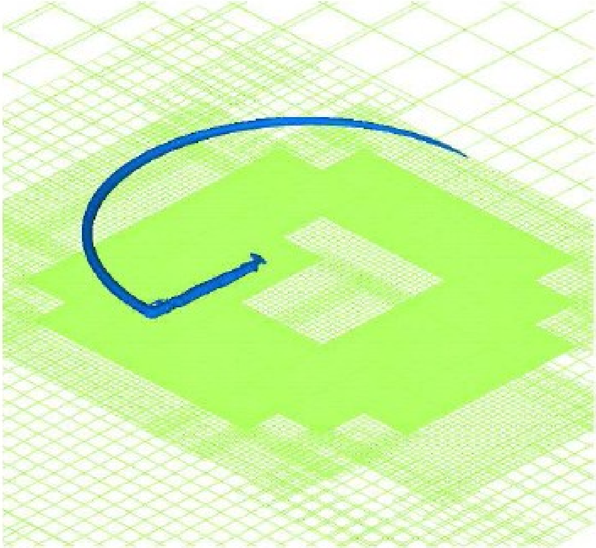
Validation: isolated blade in hover

- 7A blade in hover
- Mtip = 0.662, collective pitch = 10 degrees
- 9 remeshings following rotational of speed

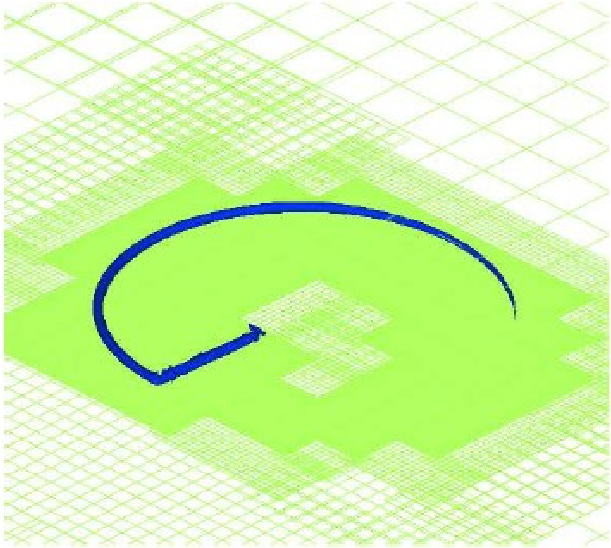


Blade mesh

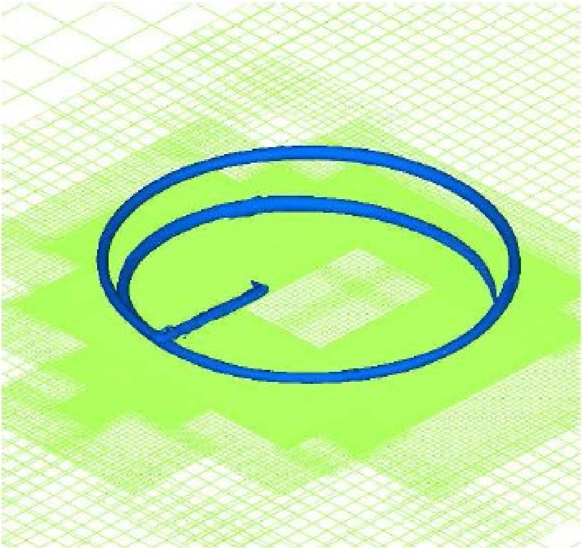
Validation: isolated blade in hover



Order 2



Order 3



Order 5 (Cartesian grids)
Order 3 (Blade grid)

Upwind schemes

- Centered schemes are well adapted for Mach > 0.2 and Mach < 1
- Extend the application domain:
  - For mach > 1, upwind Roe scheme
  - For mach < 0.2, upwind AUSM+ scheme

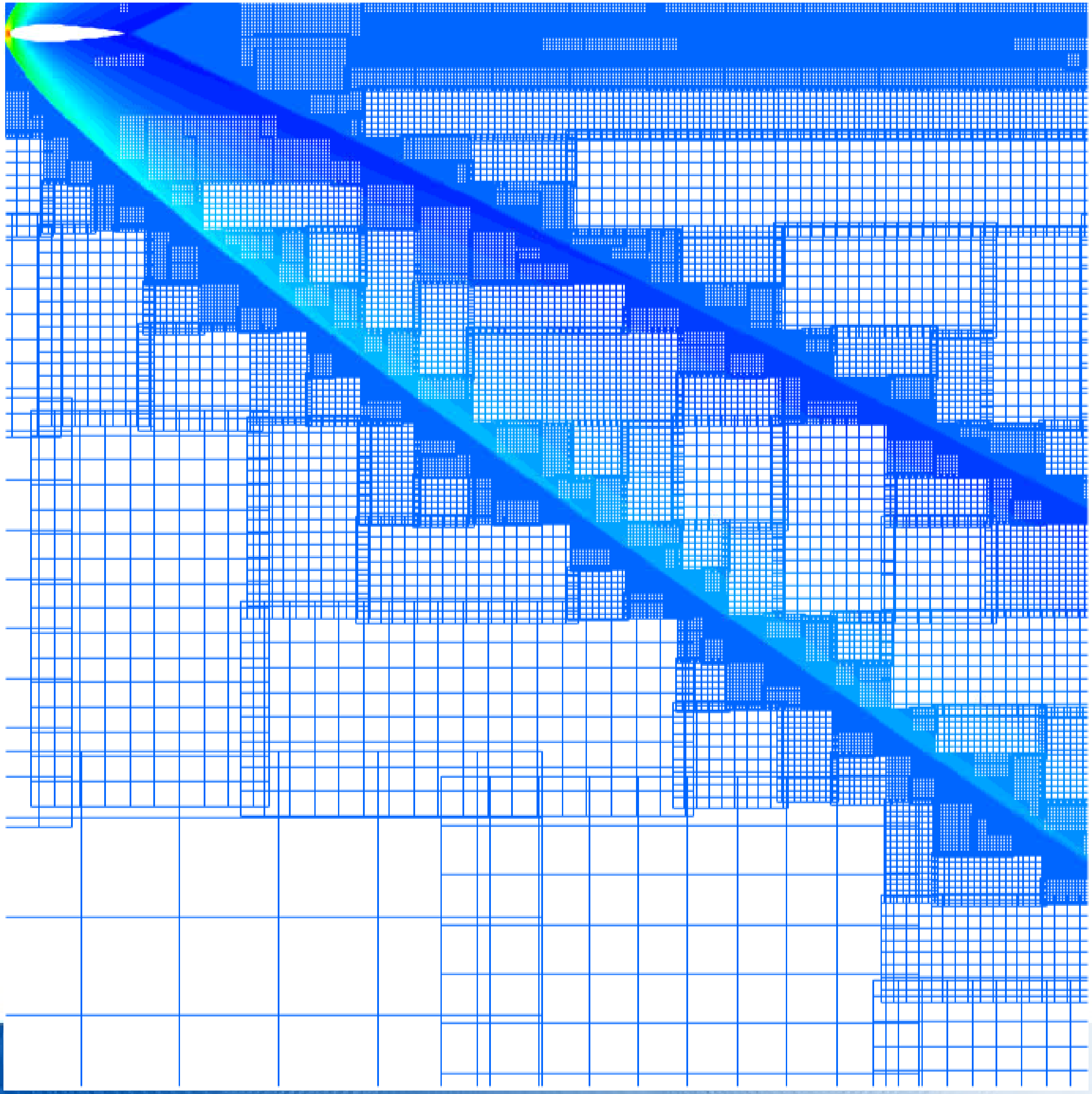
MUSCL formulation

- On Curvilinear grids:
  - Classical MUSCL approach
  - Roe and AUSM+ fluxes
- On Cartesian grids:
  - Approximate Riemann solvers can be slightly simplified
  - Weak CPU gain: 10%
  - Memory gain: 40%

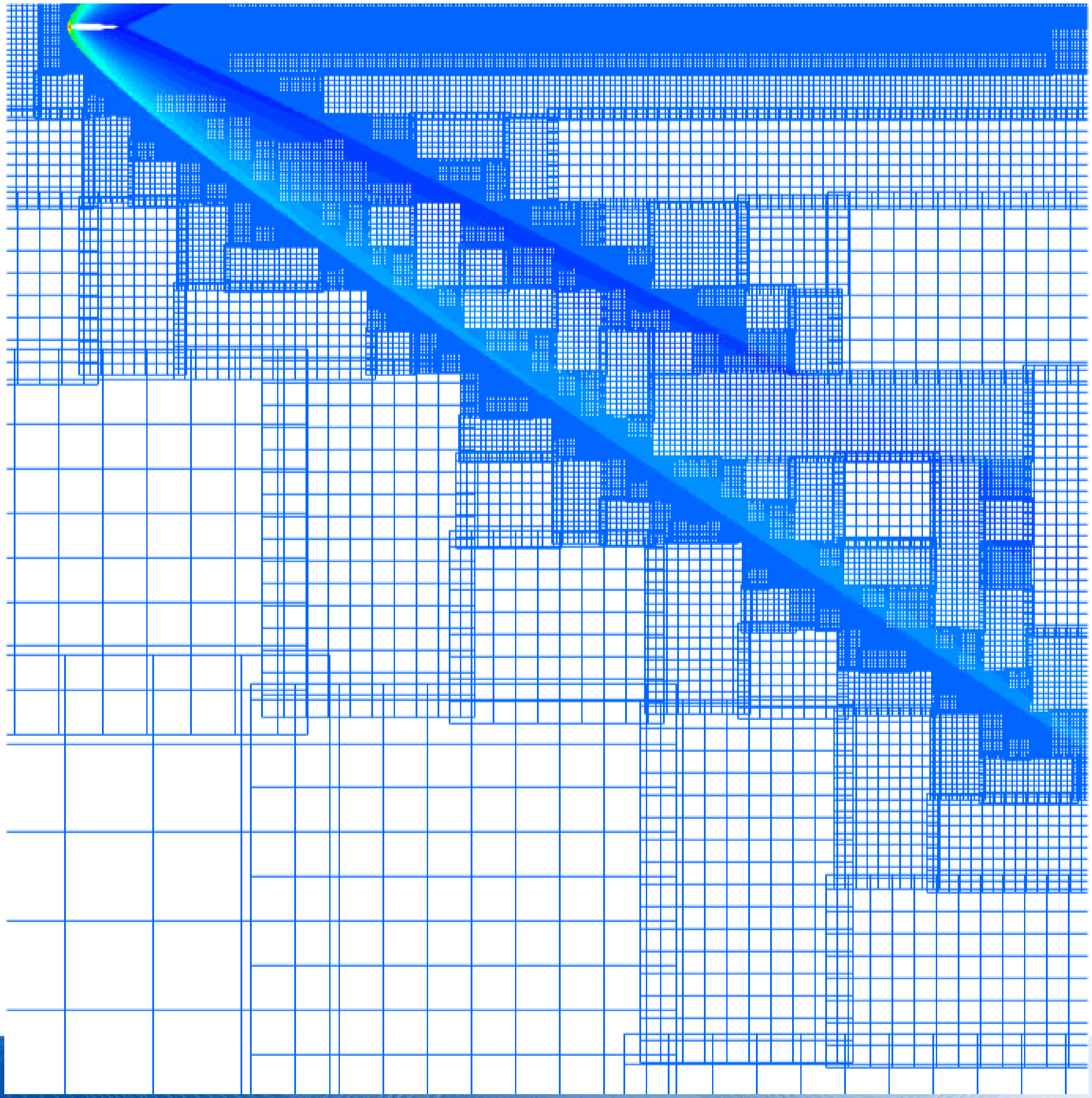
Validation: supersonic NACA0012

- NACA0012 profile at Mach 2
- Inviscid computation
- 2.4 M points after 10 mesh adaptations

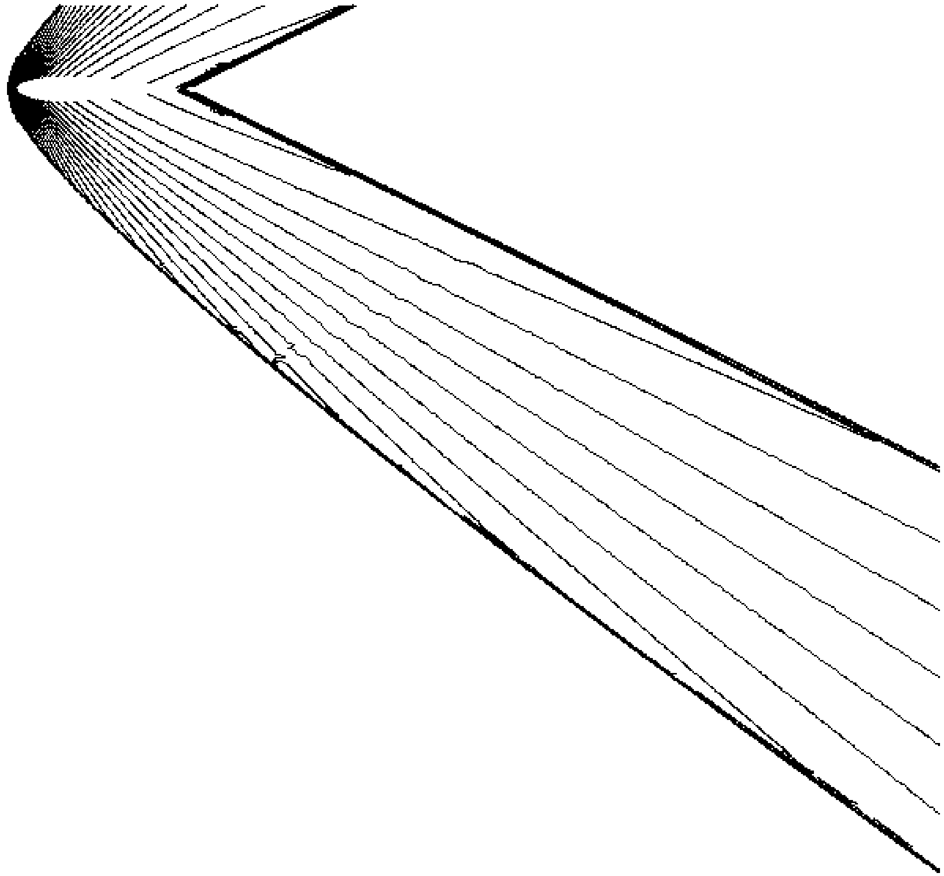
Validation: supersonic NACA0012



Validation: supersonic NACA0012

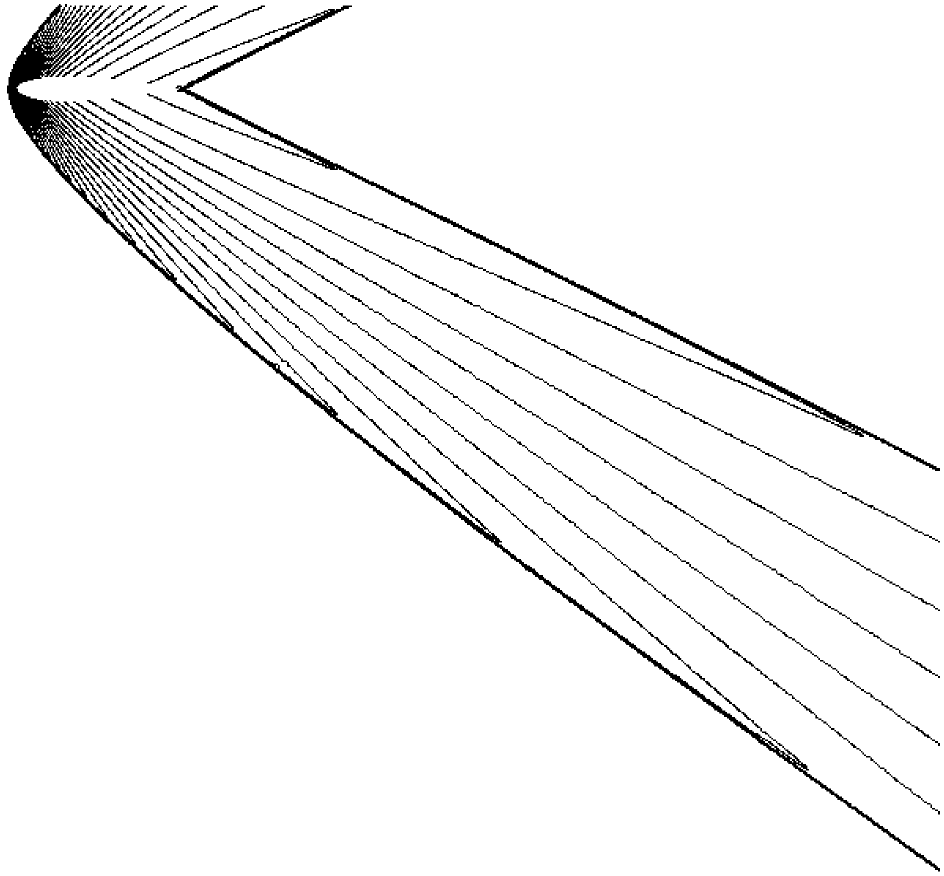


Validation: supersonic NACA0012



Jameson scheme

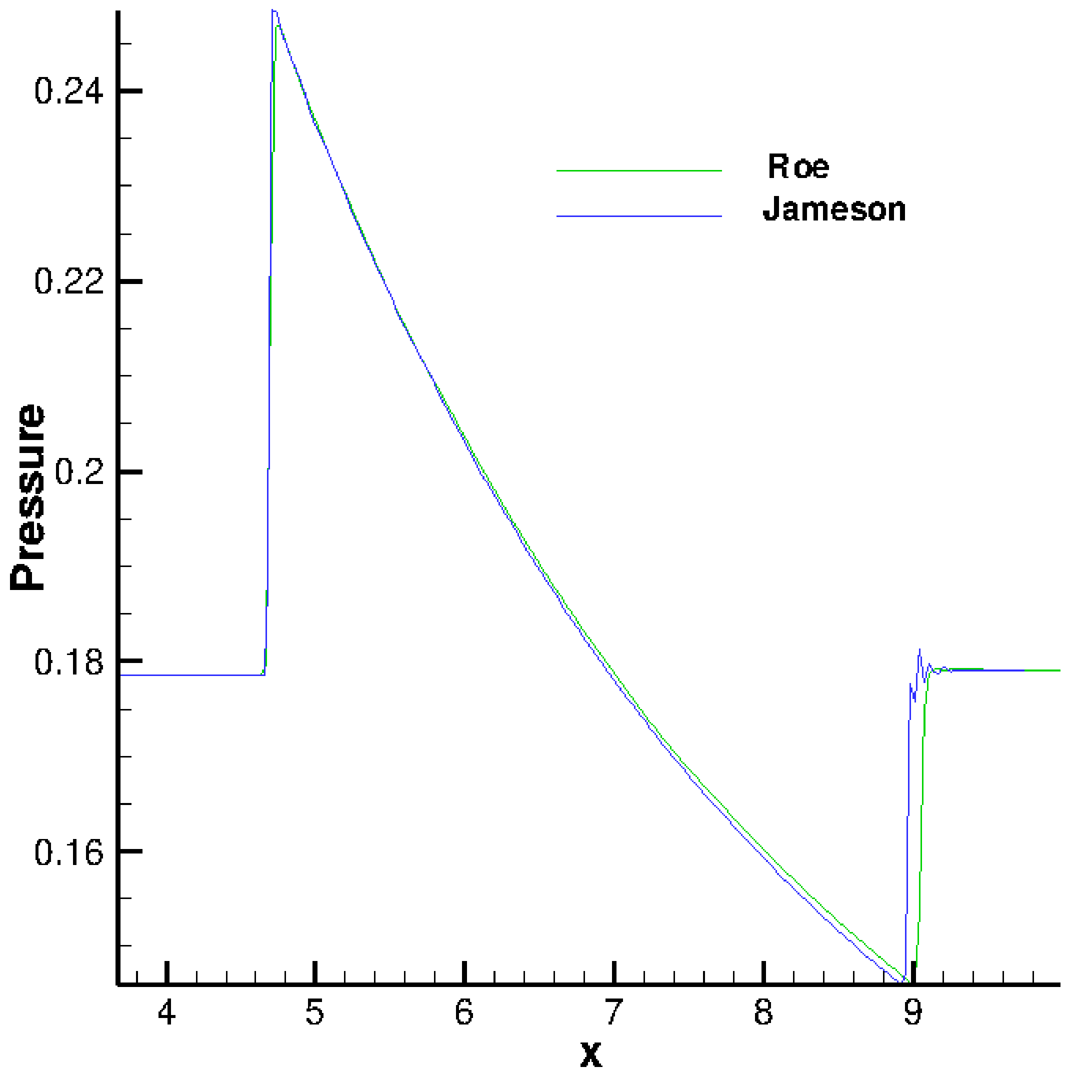
Validation: supersonic NACA0012



Roe scheme

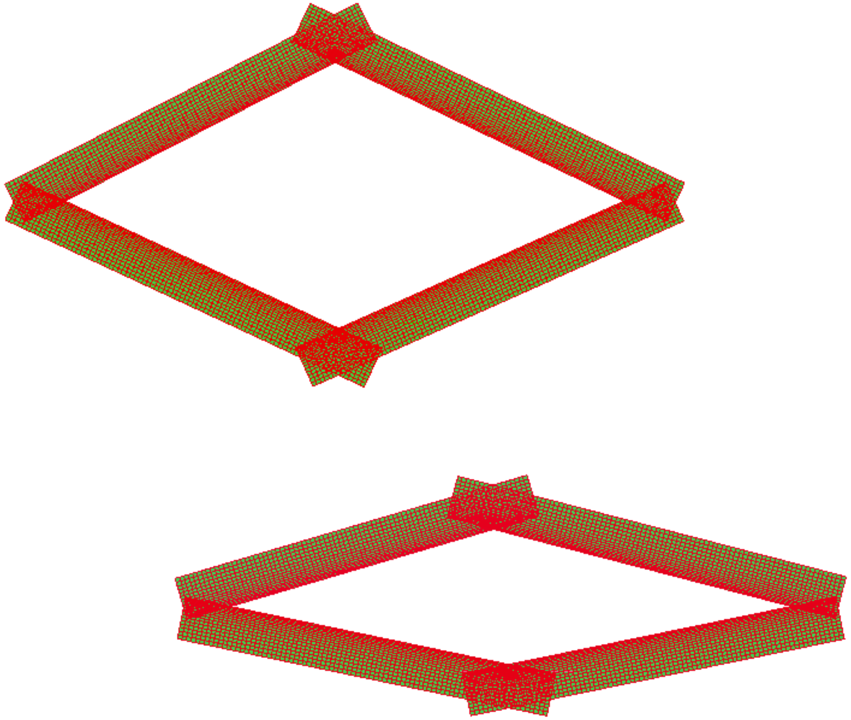
Validation: supersonic NACA0012

z=-4



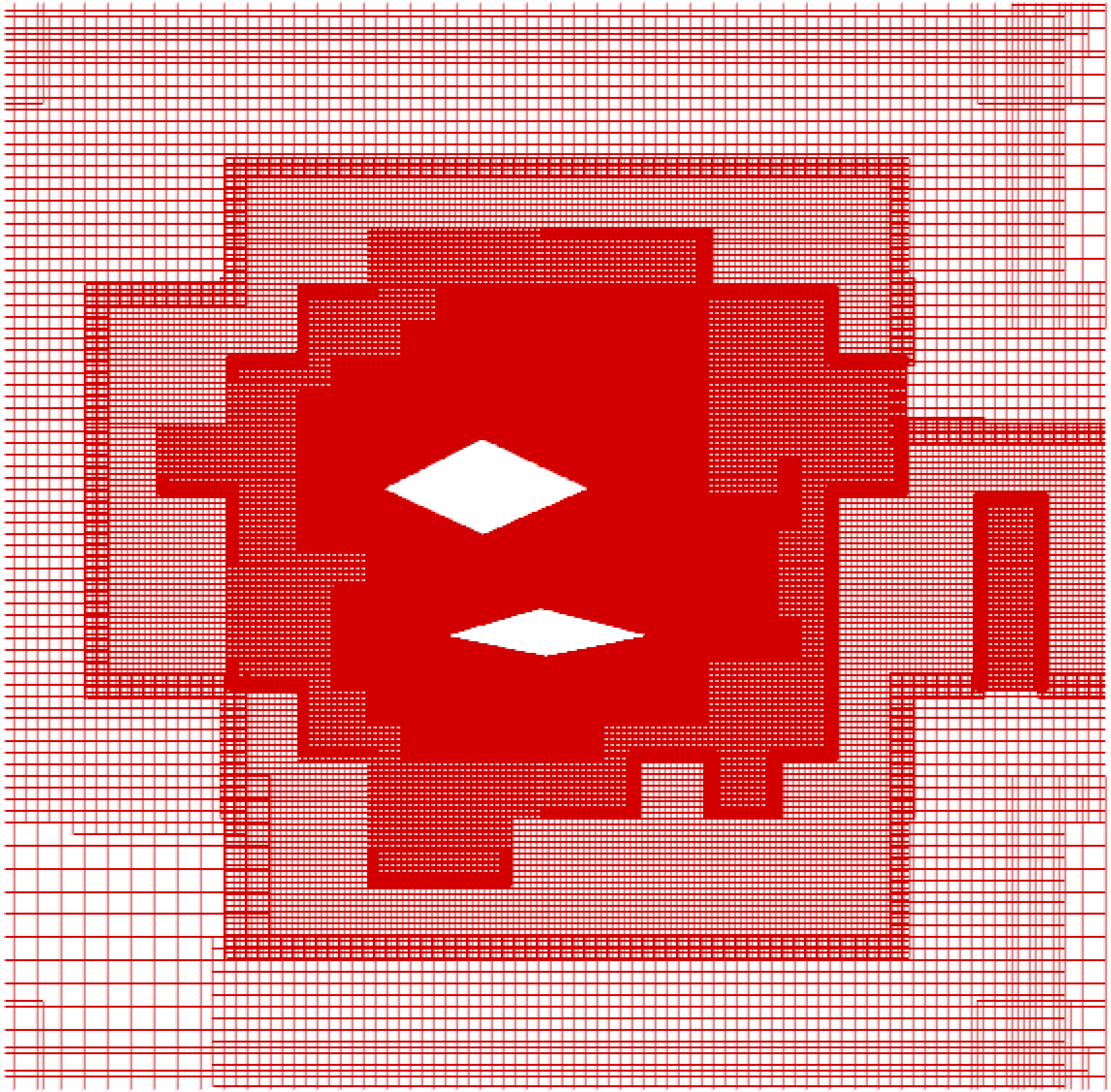
Validation: flow around two diamonds

- Two diamonds at different Mach numbers
- Body grids:

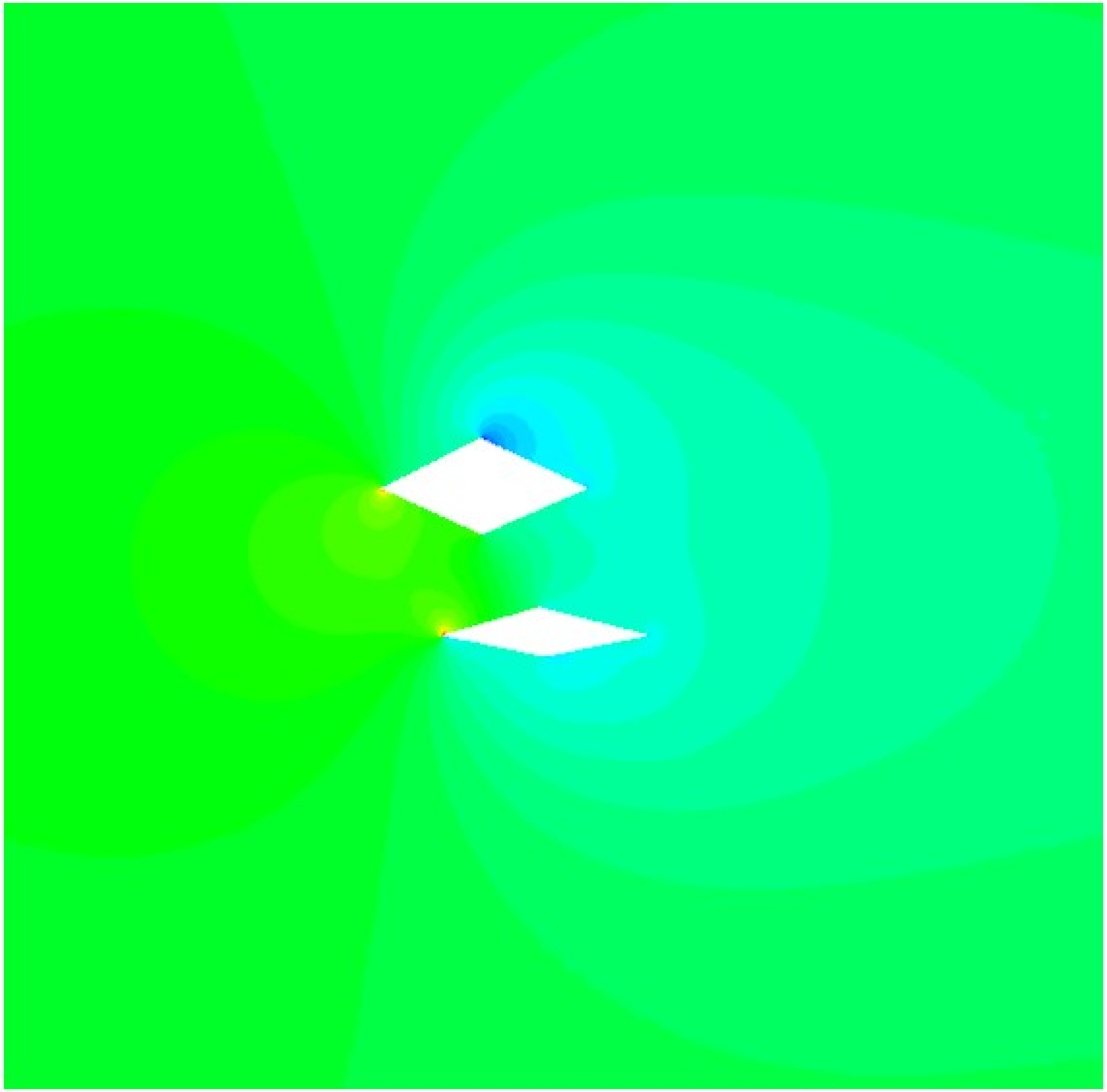


Diamonds: Mach 0.1

- AUSM+ scheme
- 5 remeshings

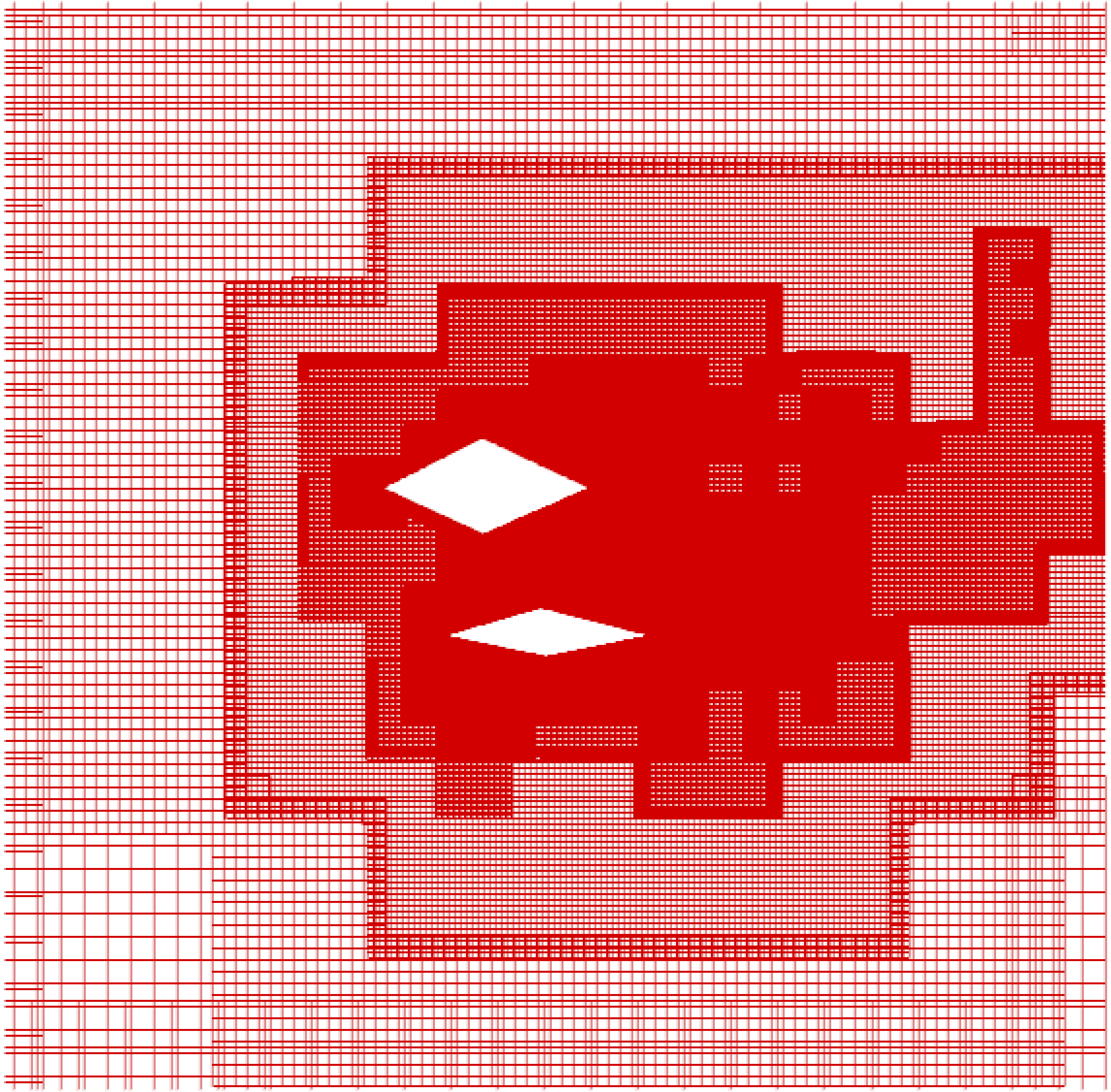


Diamonds: Mach 0.1

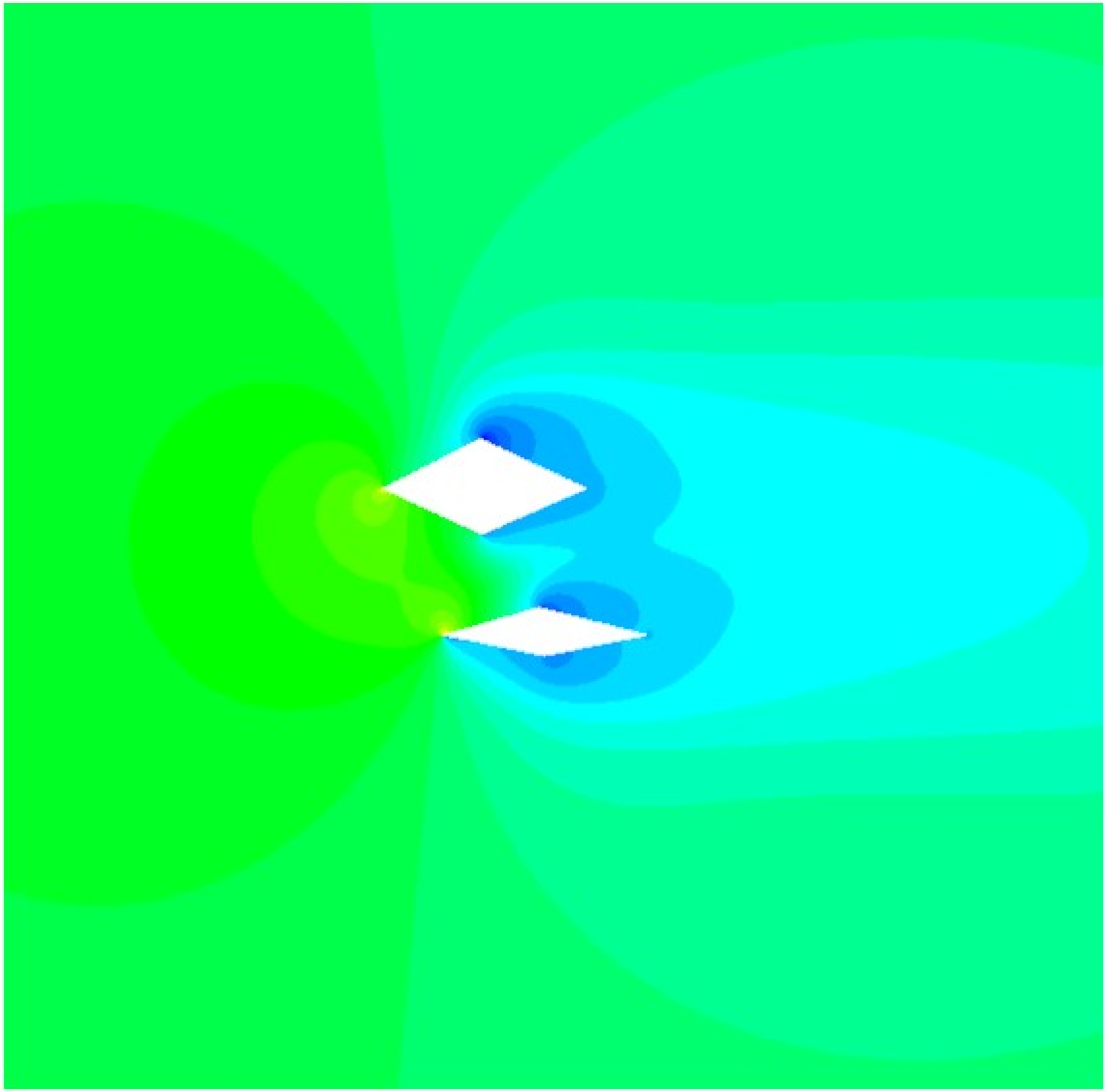


Diamonds: Mach 0.5

- Jameson scheme
- 5 remeshings

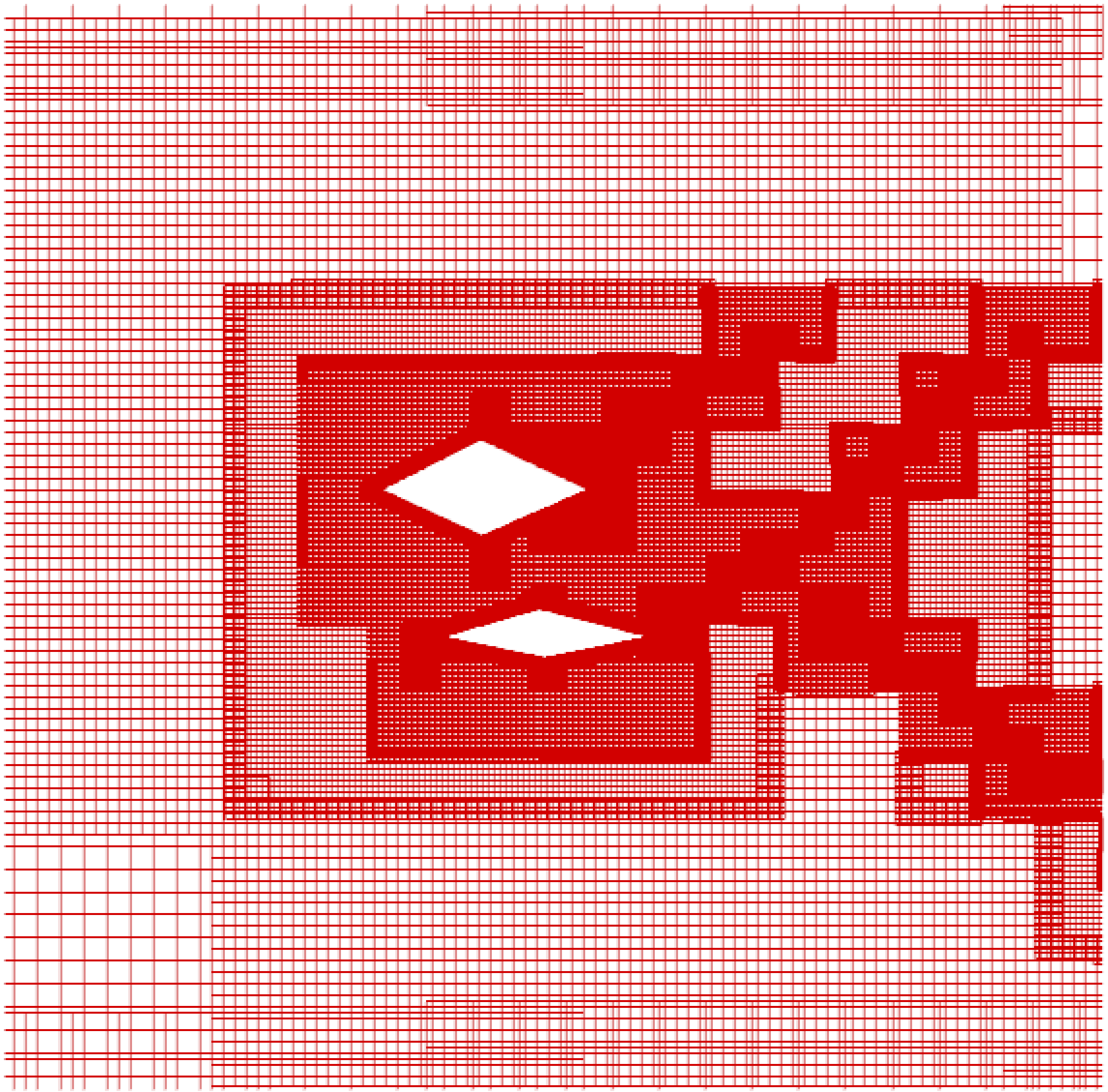


Diamonds: Mach 0.5

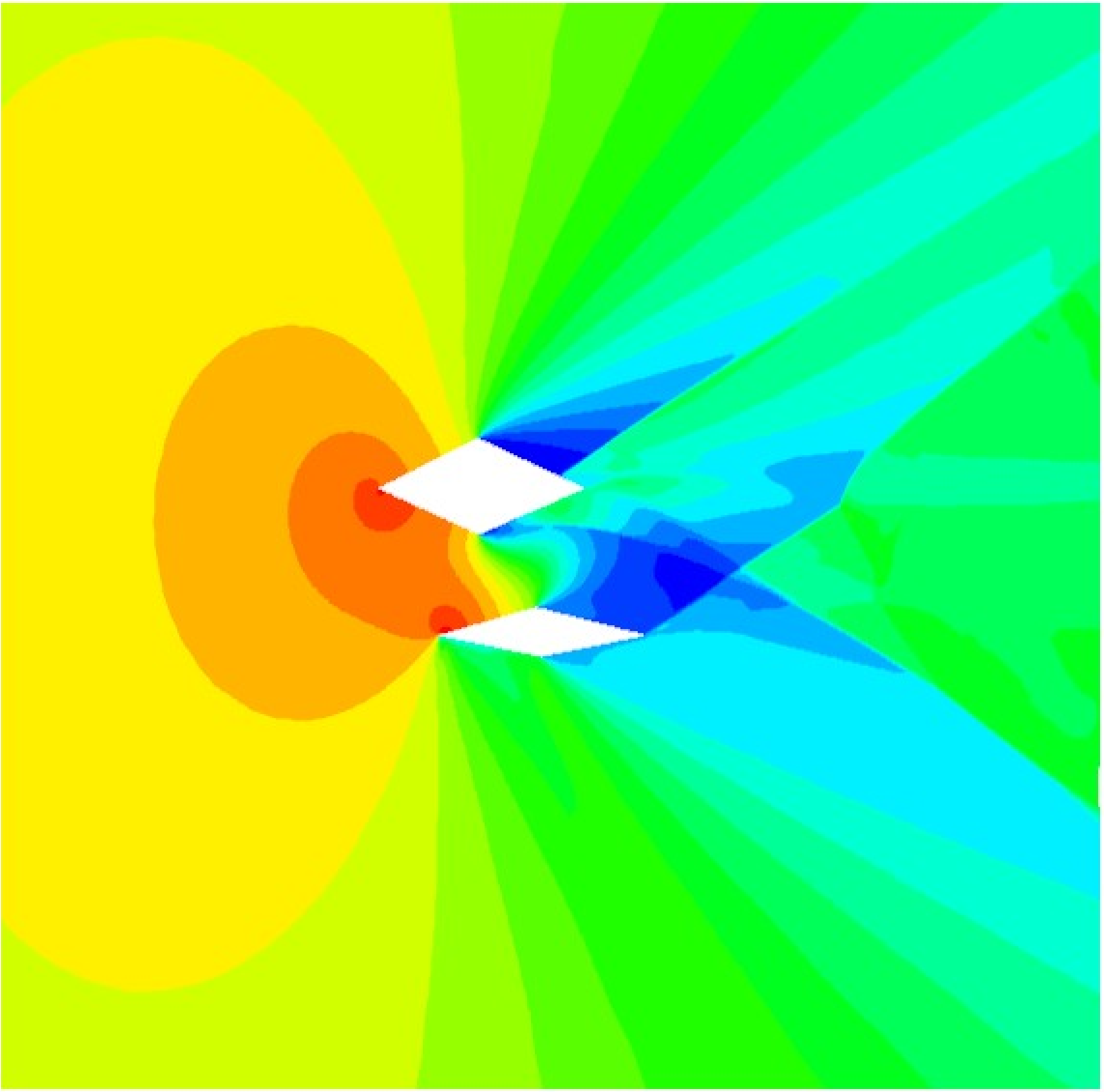


Diamonds: Mach 1.2

- Roe scheme
- 5 remeshings



Diamonds: Mach 0.5



Conclusion

- High potential of Structured Cartesian Overset grids:
  - Accuracy / efficiency of a large panel of solvers
  - No metric storage
  - Very fast interpolation cell search
  - Numerical schemes are easy to implement

- Perspective: full extension of the method to order 5